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AN INFLUENCE LINE ANALYSIS FOR SUSPENSION BRIDGES

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STRUCTURAL DIVISION

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AN INFLUENCE LINE ANALYSIS FOR SUSPENSION BRIDGES

David J. Peery,¹ M. ASCE

SYNOPSIS

Long-span suspension bridges cannot be analyzed by the usual procedures for elastic structures. Since stresses and deflections are not proportional to loads, it is not possible to superimpose various loading conditions or to use influence lines in the normal manner. It is customary to integrate the differential equation of the truss deflection curve, obtaining expressions in terms of exponential or hyperbolic functions. The equation for the cable tension component H is implicit, containing the unknown H in the exponential or hyperbolic terms. This equation must be solved by trial for numerous loading conditions.

The differential equations for a suspension bridge truss are shown here to be identical to the equations for a beam in tension. A hypothetical tension force is therefore assumed to act on the stiffening truss at all times. If the axial tension force remains constant under all loading conditions, the bending moments, shears, and deflections are directly proportional to the transverse loads, and various loading conditions may be superimposed. Influence lines are readily calculated as deflection curves of beams in tension. Influence lines for continuous trusses are obtained by superimposing deflection curves which restore continuity of slope at supports.

The ordinates of deflection curves may be readily tabulated in dimensionless form, to cover the entire range of suspension bridge proportions. From these tables, influence lines may be calculated by slide rule with no further reference to tables of exponential or hyperbolic functions. Two sets of influence lines, each corresponding to one constant value of the tension force, are calculated. Final bending moments are obtained by linear interpolation.

INTRODUCTION

Most engineering structures may be considered to be completely elastic. The structural deflections are small enough so that the moment arms of forces in the deflected structure may be assumed equal to the moment arms in the undeflected structure. These assumptions are not valid for the long-span suspension bridge, in which the stiffening truss is erected in such a manner that it resists no dead-load stress at mean temperature. The high ratio of dead load to live load and the comparative flexibility of the structure are such that the change in moment arm of the cable tension force must be considered in the analysis. The total dead- and live-load cable force is displaced to produce

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a moment increment which has a large effect on the live-load bending moment in the stiffening truss. The analysis procedure for suspension bridges which includes this effect is termed the "Deflection Theory" as distinguished from the "Elastic Theory" which follows usual structural design procedures.

The fundamental equations for the Deflection Theory were presented by Muller-Breslau in 1881 and developed by Josef Melan, in 1888. The equations were extended to a three-span suspension bridge by L. S. Moisseiff, M. ASCE, and F. E. Turneure, Hon. M. ASCE, during the design of the Manhattan Bridge in 1909. The Deflection Theory was further extended to structures of any numbers of spans, with continuous or non-continuous trusses by D. B. Steinman, M. ASCE². In all of these procedures the equation of the deflection curve is written in terms of exponential functions which contain the variable H in the exponent, and which require trial solutions using several values for the exponent. The practical calculations often require the use of six significant figures because similar large numbers must be subtracted.

It was recognized since the early days of the Deflection Theory that the differential equations are not linear, and that procedures of superposition and influence lines do not hold in the normal manner. In the conventional deflection theory, the differential equations are linearized over a small range by assuming that the exponential term containing H is constant during the integration. Exactly the same assumption is made in the influence line procedure, but engineers have been reluctant to accept the use of influence lines in deflection theory analysis.

The use of influence lines in deflection theory analysis was correctly stated and demonstrated by T. Godard³ in 1894. An extensive treatment of the deflection theory and the use of influence lines was published by Hans H. Rode⁴, M. ASCE, in 1930. Prof. Rode used a linear interpolation between two influence lines calculated for different values of H . Although Prof. Rode's paper attracted considerable attention by first introducing the effect of horizontal cable displacement, the influence line procedure has not been generally accepted in practice by American engineers.

The practical usefulness of influence line procedures has been demonstrated by Arne Selberg⁵ and S. O. Asplund⁶, M. ASCE. Prof. Selberg prepared dimensionless charts from which influence-line ordinates can be obtained for bridges of any proportions. Dr. Asplund prepared extensive dimensionless tables from which influence lines can be readily calculated for bridges of any proportions.

The use of influence lines was extended to bridges with continuous trusses by Jacob Karol⁷, M. ASCE. The moments at the towers were obtained by a

2. "A Generalized Deflection Theory for Suspension Bridges," by D. B. Steinman, Transactions, ASCE, Vol. 100, 1935, p. 1133.
3. "Recherches sur le calcul de la resistance de tabliers des ponts suspendus," by T. Godard, Annales des Ponts et Chaussées, 1894, Vol. 8, p. 105-189.
4. "New Deflection Theory," by Hans H. Rode, Transactions, Det Kongelige Norske Videnskabere Selskab, 1930, No. 3 (in English).
5. "Design of Suspension Bridges," by Arne Selberg, Det Kongelige Norske Videnskabers Selskabs Skrifter, 1945, No. 1 (in English).
6. "On the Deflection Theory of Suspension Bridges," by S. O. Asplund, Orebro, Sweden, 1943.
7. "A Partial Influence Line Procedure for Suspension Bridge Analysis by the Deflection Theory," by Jacob Karol, A Doctoral Dissertation at the University of Illinois, 1938.

procedure of moment distribution for analogous beams in tension, and dimensionless tables were compiled to facilitate analysis. Dr. Karol, in 1938, also questioned the trend toward more flexible stiffening trusses, and warned of dangers from wind action.

The usual Deflection Theory has been used for half a century, and various simplifications and short-cuts have been developed. Engineers are hesitant to adopt a new method of analysis until the computation procedure has been proven. The procedure will be explained in detail in this paper.

Beam Analogy

The assumptions commonly used in Deflection Theory analysis are retained. The initial cable curve is assumed parabolic, the moment of inertia of the stiffening truss is assumed constant in each span, and the dead load is assumed to produce no bending moment in the stiffening truss. Although other conditions may be readily considered⁸, the cable area will be assumed constant, and tower and suspender elongations will be neglected. The towers are assumed to resist no horizontal cable load.

The usual notation for suspension bridge analysis will be used⁹. The notation for general dimensions of the structure is shown in Fig. 1. Since influence line procedures are used, a single concentrated live load P is assumed at a distance k from the left end of each span.

The cable curve in each span is assumed parabolic, as defined in the following equations.

$$y = \frac{4fx}{l^2}(l-x) \quad \text{or} \quad y_i = \frac{4f_i x_i}{l_i^2}(l_i - x_i) \quad (1)$$

The suspenders are assumed to be closely spaced, and to provide a uniform live-load distribution along the span and cable of $8 f H / l^2$; in addition to a non-uniform distribution resulting from the deflections. The bending moment in the stiffening truss for any span is found from the conventional equation of the deflection theory.

$$M = M_0 - Hy - H_r \eta \quad (2)$$

The term M_0 represents the simple-beam bending moment for the live load, P . The term Hy represents the simple beam bending moment produced by the uniformly distributed suspender load of $8 f H / l^2$, where y represents the parabola defined in Eq. 1. The term H_r represents the sum of the dead-load horizontal cable tension component H_w and the live-load horizontal cable tension component H .

$$H_r = H_w + H \quad (3)$$

The moment $H_r \eta$ results from the vertical deflection η of the cable from its initial parabolic curve. This moment $H_r \eta$ represents the difference between the Moments of the Elastic Theory and the Deflection Theory, and is transmitted to the stiffening truss by means of a non-uniform suspender load on the truss.

The beam shown in Fig. 2 resists an axial force H_r , as well as the

8. Ibid., Footnote 4, preceding page.

9. Ibid., Footnote 2, preceding page.

transverse concentrated load P and the uniformly distributed load $8 f H/l^2$. The bending moment equation for this beam is also expressed by Eq. 2, when y is defined by Eq. 1, and η is the vertical deflection at any point. The suspension bridge structure is therefore directly analogous to the beam with an axial tension force H_r .

The procedure for superposition of various transverse loading conditions is well established for beams with axial loads^{10,11}. If the axial force remains constant during application of each increment of transverse load, the final deflections, shears, and bending moments may be obtained by superimposing the effects of all increments. The beam shown in Fig. 2 may therefore be analyzed by superimposing the values for the beams of Figs. 3 and 4. The bending moment, M , and the deflection, η , are expressed by the following equations.

$$M = P m_p + H m_h \quad (4)$$

$$\eta = P \eta_p + H \eta_h \quad (5)$$

The terms m_p and η_p represent bending moments and deflections of the beam shown in Fig. 3, for unit values of the transverse load P . The terms m_h and η_h represent bending moments and deflections of the beam shown in Fig. 4, for a unit value of H . The axial tension force H_r must be considered as constant in order for the superposition procedure of Eqs. 4 and 5 to be valid.

In a suspension bridge, the resultant cable force component, $H_r = H_w + H$, varies with the live load and temperature conditions. If, however, the correct final value of H_r is assumed to act for each loading condition, several loads P may be combined to yield any desired live load distribution. Further analysis will be made for a single load P in order to construct influence lines in the conventional manner.

Suspension Bridge with Simply Supported Truss Spans

The suspension bridge in which the stiffening trusses are continuous over the supports is more difficult to analyze than the bridge in which the trusses are simply supported. The equations and calculations for a simply supported case will therefore be completed first, before considering the continuous case. The equations for bending moment and deflection will first be calculated for a beam in tension. These equations will apply to any span of a suspension bridge. Where side spans are considered, subscripts may be added to the terms (l_1, f_1, x_1 , etc.) of the equations, as indicated in Fig. 1.

The bending moments and deflections for the beams of Figs. 3 and 4 are readily obtained by integrating the common beam equation.

$$\frac{d^2 \eta}{dx^2} = - \frac{M}{EI} \quad (6)$$

The following abbreviation is customary in suspension bridge analysis.

$$c^2 = \frac{H_r}{EI} = \frac{H_w + H}{EI} \quad (7)$$

10. "Aircraft Structures," by David J. Peery, McGraw Hill, 1950, p. 528.

11. "Airplane Structures," by A. S. Niles and J. S. Newell, Third Edition, John Wiley & Sons, 1943, Vol. II, p. 100.

For the beam of Fig. 3, the term containing H does not appear, and the following expressions are obtained from Eqs. 2, 6, and 7, after integrating and evaluating the constants of integration.

$$m_p = \frac{\sinh c(l-k)}{c \sinh cl} \sinh cx \quad (x < k) \quad (8)$$

$$m_p = \frac{\sinh ck}{c \sinh cl} \sinh c(l-x) \quad (x > k) \quad (9)$$

The bending moment at any point in the beam of Fig. 4 is obtained in the same manner as for the beam of Fig. 3. The term M_0 vanishes in Eq. 2, y is defined by Eq. 1, and H is assumed to have a unit value.

$$m_h = -\frac{8f}{c^2 l^2} \left(\tanh \frac{cl}{2} \sinh cx - \cosh cx + 1 \right) \quad (10)$$

The deflection resulting from a unit value of H is expressed as follows.

$$\eta_h = \frac{8f}{EIc^4 l^2} \left(\tanh \frac{cl}{2} \sinh cx - \cosh cx + 1 - \frac{c^2 l^2 y}{8f} \right) \quad (11)$$

In many calculations it is preferable to use exponential functions rather than hyperbolic functions. Equations 8, 9, 10, and 11 can be readily converted to equations containing exponential functions, by substituting the relationships $2 \sinh z = e^z - e^{-z}$ and $2 \cosh z = e^z + e^{-z}$. The hyperbolic terms are most frequently used for beams in tension, and Eqs. 8, 9, 10, and 11 are derived elsewhere¹², with slightly different notation.

In conventional elastic structures, the influence line for any redundant force is calculated as the deflection curve of the load line for a unit displacement of the redundant. The suspension bridge may also be considered as an elastic structure if H_r is assumed to have a constant value. The horizontal component H of the cable tension will be considered as the redundant force, and the influence line for H will be calculated as the deflection curve of a beam with axial tension H_r . The cable is assumed to be cut at the centerline, as shown in Fig. 5, and a unit displacement applied to the cut ends. The truss deflections, η_h/D represent ordinates of the H -influence line.

The values of the deflections η_h defined by Eq. 11 result from a unit value of H . These must be divided by D , the displacement of the redundant force H resulting from the unit value of H . The displacement D contains a term for cable stretch, L_s/AE_c , and a term for truss bending equal to the product of the suspender pull $8f/l^2$ and the deflection, η_h , summed over all spans.

$$D = \frac{L_s}{AE_c} - \sum \frac{8f}{l^2} \int_0^l \eta_h dx \quad (12)$$

The term L_s/AE_c represents the horizontal component of the cable stretch for unit H , evaluated between anchorages. The summation term includes all spans, with proper values of f and l for the side spans. The equation for the H -influence line may now be written as follows:

12. Ibid., Footnote 11, preceding page.

$$\frac{H}{P} (\text{Inf. line}) = - \frac{\eta_h}{D} \quad (13)$$

where η_h is obtained by substituting k for x in Eq. 11.

The bending moment at any point in the stiffening truss is given by Eq. 4. In order to obtain bending moment influence lines, each term will be divided by P , and all terms will be considered as functions of k .

$$\frac{M}{P} (\text{Inf. line}) = m_p (\text{Inf. line}) + m_h \frac{H}{P} (\text{Inf. line}) \quad (14)$$

The term m_p is considered as a function of k , with x constant. The term m_h is constant since it does not contain k .

Temperature stresses are calculated independently and superimposed on other stress conditions. The horizontal cable force component, H_t , resulting from a temperature change is found as follows.

$$H_t = - \frac{\omega t L_t}{D} \quad (15)$$

If the cable is cut at the centerline, the cut ends would move a distance $\omega t L_t$ for a temperature change t and a temperature coefficient ω of the cable. The term D is defined by Eq. 12. The bending moment produced by a temperature change, M_t , is found from Eq. 4, for $P = 0$.

$$M_t = H_t m_h \quad (16)$$

Numerical Application

The structure analyzed by D. B. Steinman¹³, M. ASCE, will be considered. The same structure will later be used for the continuous case. The dimensions are $l = 800$ ft, $f = 84$ ft, $l_1 = 400$ ft, and $f_1 = 21$ ft. The truss stiffness is the same in all spans, $E I = E I_1 = 56,840,000$ ft² kips. For the cable, $L_s = 2075$ ft, $L_t = 1998$ ft, $A = 87.8$ in.², and $E_c = 25,000$ kips per sq. in. The loading per cable is 3850 lb per ft dead load (w) and 1300 lb per ft live load. The dead load cable component $H_w = w l^2 / 8 f$ is 3667 kips. Other values are $t = \pm 60^\circ$ F, $\omega = 0.0000065$, $8 f / l^2 = 8 f_1 / l_1^2 = 1 / 952.4$.

Two sets of influence lines will be constructed. One set for $c/l = 6.0$ and $c_1/l_1 = 3.0$ corresponds approximately to the minimum value of H_r . The other set, for $c/l = 7.0$ and $c_1/l_1 = 3.5$, corresponds to a value of H_r somewhat less than the maximum value. Final bending moments will be obtained by linear interpolation between values for the two sets of influence lines.

The H-influence line will first be calculated from Eqs. 12 and 13. These equations require values of η_h from Eq. 11. The values of η_h are shown in Table 1. The bending moments m_h resulting from a unit value of H are also shown in Table 1. The values shown in Table 1 are dimensionless, and apply to any structure having the proper value of c/l . The values in the last two lines of Table 1 are obtained by integrating and differentiating Eq. 11, and will be used for calculating D and for the later analysis of the continuous truss.

The value of D is obtained from Eq. 12. The integrals of Eq. 12 are evaluated as follows for the side span, with $c/l = 3.0$

$$- \frac{8 f_1}{l_1^2} \int_0^{l_1} \eta_h dx = \frac{8 f_1}{l_1^2} \frac{f_1 l_1^3}{E I_1 x 10^4} (349.1) = 0.000867 \text{ ft/kip}$$

13. Ibid., Footnote 2, page 2.

For the main span, with $cl = 6.0$,

$$-\frac{8f}{f^2} \int_0^l h \, dx = \frac{8f}{f^2} \frac{ff^3}{E I x 10^4} (143.9) = 0.011433 \text{ ft/kip}$$

The cable term is $L_S/A E_C = 0.000945 \text{ ft/kip}$ and D is obtained as the sum of terms for the main span and two side spans.

$$D = 2(0.000867) + 0.011433 + 0.000945 = 0.014112 \text{ ft/kip}$$

Similar calculations are made for $cl = 7.00$ and $c_1 l_1 = 3.50$.

$$D = 0.000945 + 2 \frac{8f_1}{f_1^2} \frac{f_1 f_1^3}{E I_1 x 10^4} (297.9) + \frac{8f}{f^2} \frac{ff^3}{E I x 10^4} (112.2) =$$

0.011342 ft/kip . The influence lines for H are calculated in columns 2 and 6 of Table 2, using Eq. 13. The values of η_h are listed in dimensionless form in Table 1, and must be multiplied by $f^2/(E I D x 10^4)$ to obtain the influence line ordinates. For the side span with $cl = 3.00$, values in column 4 of Table 1 (abbreviated $\textcircled{4}$) must be multiplied by

$$\frac{f_1 f_1^2}{E I_1 D x 10^4} = \frac{4.189}{10^4} . \text{ For the main span with } cl = 6.0 \text{ values in column 6 of}$$

Table 1 are multiplied by $67.02 x 10^{-4}$.

The influence lines for bending moment in the main span at a point $0.2l$ from the support are calculated in Table 2. Values of $-m_h/f$ for this point are given in Table 1 as 0.1536 for $cl = 6.0$ and 0.1225 for $cl = 7.0$. For $f = 84 \text{ ft}$, the values of m_h are -12.90 and -10.29 , and these values are used as multipliers of the ordinates of the H -influence lines to obtain columns 3 and 7 of Table 2. Values of m_p are calculated from Eqs. 8 and 9, and tabulated in Columns 4 and 8. The final bending moment influence lines are calculated in columns 5 and 9, by adding values of m_p and $H \cdot m_h$ as specified in Eq. 14.

The temperature effects on H and M are calculated from Eqs. 15 and 16. For $cl = 6.00$, $H_t = \pm 55.2 \text{ kips}$, $M_t = \pm 55.2 x 12.90 = \pm 712 \text{ ft-kips}$. For $cl = 7.00$, $H_t = \pm 68.7 \text{ kips}$, $M_t = \pm 68.7 x 10.29 = \pm 707 \text{ ft-kips}$. The temperature effects are superimposed on the live-load effects.

The areas under the influence lines may be calculated readily by Simpson's rule. Comparison with areas obtained by integration indicates five-place accuracy of Simpson's rule for the shapes of the influence lines. Values are shown in Table 3 for a live load extending over four-tenths of the main span, and a temperature rise of 60° F . Calculations were also made by the customary deflection theory for comparison. The only error introduced by the influence line procedure comes from the linear interpolation. The deflection theory calculations were made for $cl = 6.70$. If influence lines are calculated for this value, the final results are identical. The linear interpolation yields bending moments which are 0.2 percent greater than those calculated in the usual manner. The calculations were carried out to five and six significant figures for the comparison, although values in Tables 1 and 2 were rounded off. Normally, sliderule calculations are adequate, except for calculating m_h , η_h , and m_p from the hyperbolic functions.

The values of terms shown in Table 1, and values of m_p/f may be tabulated for all bridge proportions. The author¹⁴ has prepared dimensionless tables

14. "A Deflection Theory Analysis of Suspension Bridges by the Use of Influence Lines," by David J. Peery, A Doctoral Dissertation at the University of Michigan, 1941.

Table 1
TRUSS MOMENTS AND DEFLECTIONS
RESULTING FROM A UNIT VALUE OF H

1	2	3	4	5	6	7
$\frac{x}{l}$	$-\frac{m_h}{f}$ (Eq. 10)		$-\frac{\eta_h E I \times 10^4}{f l^2}$ (Eq. 11)			
	$c f = 6.0$	$c f = 7.0$	$c f = 3.0$	$c f = 3.5$	$c f = 6.0$	$c f = 7.0$
0	0	0	0	0	0	0
.05	.0573	.0482	88.0	74.9	36.9	28.9
.10	.0995	.0819	172.6	147.6	72.4	56.8
.15	.1307	.1057	252.4	215.8	105.4	82.5
.20	.1536	.1225	325.1	277.8	135.1	105.6
.25	.1703	.1341	389.2	332.3	161.0	125.7
.30	.1822	.1421	443.4	378.4	182.7	142.4
.35	.1906	.1475	436.5	415.0	199.8	155.6
.40	.1961	.1509	517.9	441.5	212.2	165.1
.45	.1991	.1528	536.9	457.6	219.7	170.9
.50	.2001	.1534	543.3	463.1	222.2	172.8
AREA UNDER CURVES						
$-\frac{E I \times 10^4}{f l^3} \int_0^l \eta_h dx$			349.1	297.9	143.9	112.2
SLOPE AT SUPPORTS						
$\left(\frac{d \eta_h}{dx}\right)_0 \times \frac{E I}{f l} \times 10^4$			1762	1509	742.6	583.5

for $c f = 0.5, 1.0, 1.5$ ----10.0. The use of such tables eliminates the need of any other tables, and all further influence line calculations may be made by sliderule.

Suspension Bridge with Continuous Stiffening Truss

The suspension bridge in which the stiffening trusses are continuous through the towers has additional redundancies. The truss bending moments at the towers, T_1 and T_2 , must be sufficient to provide equal slopes of the truss deflection curves for spans adjacent to the towers. The effects of these moments may be superimposed on other effects when H_r is constant.

The effect of the end moment T shown in Fig. 6 is readily calculated from the beam equations.

Table 2
INFLUENCE LINES FOR H AND FOR
BENDING MOMENT AT 0.2l

c/l = 6.00, c ₁ l ₁ = 3.00					c/l = 7.00, c ₁ l ₁ = 3.50			
1	2	3	4	5	6	7	8	9
k f	H-Inf. Line	H x m _h	m _p	M	H Inf. Line	H x m _h	m _p	M
SIDE								
SPAN ④	x 4.189	② x -12.90	Eqs. 8 & 9		⑤ x 5.212	③ x -10.29	Eqs. 8 & 9	
0	0	0	0	0	0	0	0	0
.1	.072	- .93	0	- .93	.077	- .79	0	- .79
.2	.136	- 1.75	0	- 1.75	.145	- 1.49	0	- 1.49
.3	.182	- 2.35	0	- 2.35	.197	- 2.03	0	- 2.03
.4	.217	- 2.80	0	- 2.80	.230	- 2.37	0	- 2.37
.5	.228	- 2.94	0	- 2.94	.241	- 2.48	0	- 2.48
MAIN								
SPAN ⑥	x 67.02				⑦ x 83.39			
0	0	0	0	0	0	0	0	0
.05	.247	- 3.19	12.22	9.03	.241	- 2.48	10.07	7.59
.10	.495	- 6.39	25.57	19.18	.474	- 4.88	21.38	16.50
.15	.706	- 9.11	41.22	32.11	.688	- 7.08	35.34	28.26
.20	.905	-11.67	60.62	48.95	.881	- 9.07	53.67	44.60
.25	1.079	-13.92	44.90	30.98	1.048	-10.78	37.82	27.04
.30	1.224	-15.79	33.26	17.47	1.187	-12.21	26.65	14.44
.35	1.339	-17.27	24.63	7.36	1.298	-13.36	18.78	5.42
.40	1.422	-18.34	18.25	- .09	1.377	-14.17	13.23	- .94
.45	1.472	-18.99	13.50	- 5.49	1.425	-14.66	9.32	- 5.34
.50	1.489	-19.21	9.99	- 9.22	1.441	-14.83	6.57	- 8.26
.55	1.472	-18.99	7.39	-11.60	1.425	-14.86	4.62	-10.04
.60	1.422	-18.34	5.45	-12.89	1.377	-14.17	3.25	-10.92
.65	1.339	-17.27	4.01	-13.26	1.298	-13.36	2.28	-11.08
.70	1.224	-15.79	2.94	-12.85	1.187	-12.21	1.60	-10.61
.75	1.079	-13.92	2.12	-11.80	1.048	-10.78	1.11	- 9.67
.80	.905	-11.67	1.50	-10.17	.881	- 9.07	.75	- 8.32
.85	.706	- 9.11	1.02	- 8.09	.688	- 7.08	.53	- 6.55
.90	.495	- 6.39	.64	- 5.75	.474	- 4.88	.30	- 4.58
.95	.247	- 3.19	.30	- 2.89	.241	- 2.48	.14	- 2.34
1.00	0	0	0	0	0	0	0	0

$$M = -EI \frac{d^2\eta}{dx^2} = \frac{T_x}{l} - H_r \eta \quad (17)$$

Integrating, and substituting the boundary conditions $M = \eta = 0$ when $x = 0$, and $M = T$, $\eta = 0$, when $x = l$, the following equations are obtained.

$$M = T \frac{\sinh cx}{\sinh cl} \quad (18)$$

$$\eta = \frac{T}{H_r} \left(\frac{x}{l} - \frac{\sinh cx}{\sinh cl} \right) \quad (19)$$

Table 3
COMPARISON OF RESULTS WITH VALUES
FROM COMMON DEFLECTION THEORY

	1	2	3	4	5
Values of H	Influence Line Solution			Def. Theory	Percent Error
	$cf = 6.00$	$cf = 7.00$	$cf = 6.70$ Interpo- lation	$cf = 6.70$	
Live Load, 0.4 span	349.3	339.0			
Temperature + 60°	-55.2	-68.7			
L. L. + Temp.	294.1	270.3	277.5	277.5	0.
Values of M					
Live Load, 0.4 span	8477	7339			
Temperature + 60°	712	707			
L. L. + Temp.	9189	8046	8389	8371	0.2%

If the moment T is applied at the left end of the span, the term x in Eqs. 18 and 19 is replaced by $l - x$.

The influence line for H will be a deflection curve for a unit displacement of H , similar to that shown in Fig. 5, except that corrections are made for continuity. The deflections η_h resulting from a unit value of H are shown in Fig. 7(a) for simply supported spans. These yield an angular discontinuity ϕ_h at each tower. The tower moments T_h shown in Fig. 7(b) are sufficient to remove the angular discontinuity at the towers for the unit value of H , and produce deflections η_{ht} . The H -influence line is now obtained in Fig. 7(c) by superimposing deflections η_h and η_{ht} to yield deflections of the continuous beam for a unit H , then dividing by D_1 , the displacement of the redundant H in a continuous bridge for a unit H . The term D_1 is similar to the term D for the noncontinuous bridge (Eq. 12), but also includes the continuity effects η_{ht} .

$$D_1 = \frac{L_s}{AE_c} - \sum \frac{\partial f}{\partial H} \int_0^l \eta_h dx - \sum \frac{\partial f}{\partial H} \int_0^l \eta_{ht} dx \quad (20)$$

The H -influence line is obtained from the following equation.

$$\frac{H}{P} (\text{Inf. line}) = - \frac{\eta_h + \eta_{ht}}{D_1} \quad (21)$$

The influence line for the bending moment at any point in a continuous suspension bridge will be calculated from the influence line for the bending moment in a continuous beam with axial tension H_r , shown in Fig. 8(a). The influence line for bending moment at point x is a deflection curve for unit angular displacement at point x , as shown in Fig. 8(d). The influence line ordinates m_p for a non-continuous beam were calculated by Eqs. 8 and 9, and have slopes ϕ_{p1} and ϕ_{p2} at the towers, as shown in Fig. 8(b). Tower moments, T_{p1} and T_{p2} , must be applied to remove the angular rotations, as shown in Fig. 8(c). The deflections m_{tp} resulting from these moments are superimposed on the values of m_p in order to correct the influence lines for continuity

effects. The final ordinates of the bending moment influence line for the continuous beam in tension are $m_p + m_{tp}$ as shown in Fig. 8(d).

The final bending moment influence line in a continuous suspension bridge is obtained by superimposing the cable effect, $H(m_h + m_{th})$, on the continuous beam effect.

$$\frac{M}{P} (\text{Inf. line}) = m_p + m_{tp} + (m_h + m_{th}) \frac{H}{P} (\text{Inf. line}) \quad (22)$$

This equation differs from Eq. 14 by the inclusion of continuity effects m_{tp} and m_{th} .

Applications to Bridge with Continuous Stiffening Trusses

The structure which was previously analyzed as a bridge with simply supported spans will now be considered as having continuous spans. Calculations will be shown for only one set of influence lines, for $c_l = 7.00$ and $c_1 l_1 = 3.50$. Similar calculations for $c_l = 6.00$ and $c_1 l_1 = 3.00$ must be made, but only the results are included.

The deflections in the side span for a moment T_1 at the tower are calculated from Eq. 19, and shown in dimensionless form in Column 2 of Table 4. Similar calculations for the main span are shown in Columns 7 and 8 for moments at each tower. Deflections for equal moments $T = T_1 = T_2$ at the towers are calculated in Column 9 as the sum of terms in Columns 7 and 8. The areas of these deflection curves and their slopes at the towers are shown at the bottoms of the columns.

The end moments T_h for a unit value of H will be evaluated by equating the angles at the towers. From Table 1,

$$\phi_h = 0.1509 \frac{f_1 l_1}{EI_1} + 0.07426 \frac{f l}{EI} \quad (23)$$

The tower moments T_h must produce an equal angular deflection, as obtained from Table 4.

$$\phi_h = 2.506 \frac{T_h}{H_r l_1} + 6.987 \frac{T_h}{H_r l} \quad (24)$$

Equating the angles of Eqs. 23 and 24

$$\frac{T_h}{H_r} = 0.006434 \frac{f l^2}{EI} \quad (25)$$

The last term in Eq. 20 will be calculated from this value of T_h/H_r and from the areas shown in Table 4.

$$-\int \frac{\partial f}{\partial z^2} \eta_{ht} dx = -2 \times \frac{\partial f}{\partial z^2} \times 0.2310 \frac{T_h l_1}{H_r} - \frac{\partial f}{\partial z^2} \times 0.7148 \frac{T_h l}{H_r} = -0.004835 \quad (26)$$

The value of D_1 is now obtained by adding this term to the value of D previously calculated for the non-continuous structure.

$$D_1 = 0.011341 - 0.004835 = 0.006506 \text{ ft/kip}$$

This value of D_1 and the value of T_h/H_r are used to calculate values of η_{ht}/D_1 in Columns 3 and 10 by multiplying the dimensionless values of Columns 2 and

Table 4
INFLUENCE LINE FOR H

Side Span $c/l = 3.50$					Main Span $c/l = 7.00$						
1	2	3	4	5	6	7	8	9	10	11	12
k	η	η_{ht}	η_h	H	k	η	η	η	η_{ht}	η_h	H
\bar{l}	\bar{T}_1/H_r	\bar{D}_1	\bar{D}_1	\bar{P}	\bar{l}	\bar{T}_2/H_r	\bar{T}_1/H_r	\bar{T}/H_r	\bar{D}_1	\bar{D}_1	\bar{P}
	Eq. 19	-0.935②	9.085⑤	Inf. Line ③+④		Eq. 19	Eq. 19	⑦+⑧	-0.935⑨	145.6⑩	Inf. Line ⑩+⑪
0	0	0	0	0	0	0	0	0	0	0	0
.1	.074	-.073	.134	.061	.05	.049	.245	.294	-.276	.421	.145
.2	.154	-.144	.152	.108	.10	.099	.403	.502	-.470	.825	.355
.3	.224	-.210	.344	.134	.15	.148	.500	.648	-.606	1.199	.593
.4	.285	-.266	.401	.135	.20	.197	.553	.750	-.702	1.536	.834
.5	.331	-.310	.421	.111	.25	.245	.576	.821	-.768	1.827	1.059
.6	.357	-.334	.401	.067	.30	.293	.577	.870	-.814	2.071	1.257
.7	.352	-.330	.344	.014	.35	.339	.564	.903	-.845	2.262	1.417
.8	.305	-.285	.252	-.033	.40	.385	.539	.924	-.864	2.400	1.536
.9	.196	-.183	.134	-.049	.45	.429	.507	.936	-.876	2.484	1.608
1.0	0	0	0	0	.50	.470	.470	.940	-.879	2.512	1.633
ϕ \bar{T}/H_r	2.506					.987	6.00	6.987			
Area \bar{T}/H_r	.2310					.3574	.3574	.7148			

Table 5
INFLUENCE LINE FOR BENDING MOMENT AT 0.2l

Side Spans $m_p = 0$				Main Span					
1	2	3	4	5	6	7	8	9	10
k \bar{l}_1	$\frac{H}{P}(m_h + m_{th})$	m_{tp}	$\frac{M}{P}$	k \bar{l}	$\frac{H}{P}(m_h + m_{th})$	m_p	m_{tlp}	m_{t2p}	$\frac{M}{P}$
			Inf. Line ②+③						Inf. Line
Left	-3.67 ⑤	-18.03 ②			-3.67 ①②	Eqs. 8 & 9	-18.03 ⑧	1.37 ⑦	
0	0	0	0	0	0	0	0	0	0
.1	-.22	-1.33	-1.55	.05	-.53	10.07	-4.42	.07	5.19
.2	-.40	-2.78	-3.18	.10	-1.30	21.38	-7.28	.13	12.93
.3	-.49	-4.04	-4.53	.15	-2.18	35.34	-9.03	.20	24.33
.4	-.49	-5.14	-5.63	.20	-3.06	53.67	-9.98	.27	40.90
.5	-.41	-5.96	-6.37	.25	-3.88	37.82	-10.39	.34	23.89
.6	-.25	-6.47	-6.72	.30	-4.61	26.65	-10.41	.40	12.03
.7	-.05	-6.35	-6.40	.35	-5.20	18.78	-10.16	.46	3.88
.8	+.12	-5.50	-5.38	.40	-5.64	13.23	-9.72	.51	-1.62
.9	+.18	-3.54	-3.36	.45	-5.90	9.32	-9.15	.59	-5.14
1.0	0	0	0	.50	-6.00	6.57	-8.47	.64	-7.36
Right		1.37 2		.55	-5.90	4.62	-7.74	.70	-8.32
0	0	0	0	.60	-5.64	3.25	-6.95	.74	-8.60
.1	+.18	.27	.09	.65	-5.20	2.28	-6.13	.77	-8.28
.2	+.12	.42	.30	.70	-4.61	1.60	-5.27	.79	-7.49
.3	-.05	.48	.43	.75	-3.88	1.11	-4.42	.79	-6.40
.4	-.25	.49	.24	.80	-3.06	.75	-3.54	.76	-5.09
.5	-.41	.45	.04	.85	-2.18	.53	-2.66	.69	-3.62
.6	-.49	.39	-.10	.90	-1.30	.30	-1.78	.55	-2.23
.7	-.49	.31	-.18	.95	-.53	.14	-.89	.34	-.94
.8	-.40	.21	-.19	1.0	0	0	0	0	0
.9	-.22	.10	-.12						
1.0	0	0	0						

9 by $T_h/H_r D_1 = 0.9353$. The values of $-\eta_h/D_1$ shown in Columns 4 and 11 are calculated from Table 1 in the same manner as for the non-continuous structure previously analyzed. The H-influence line ordinates are now obtained in Columns 5 and 12 of Table 4 by summing the preceding columns.

The influence line for bending moment at $x = 0.2l$ is calculated in Table 5. The bending moment m_{th} , the continuity correction to the bending moment for a unit value of H, is calculated from Eq. 18 by substituting T_h from Eq. 25. Since T_h acts at both ends of the main span, values of $x = 0.2l$ and $l - x = 0.8l$ must both be used.

$$m_{th} = T_h \frac{\sinh 0.2 cl + \sinh 0.8 cl}{\sinh cl} = 6.62$$

The value of $m_h = -10.29$ is the same as previously calculated for the non-continuous structure.

$$m_h + m_{th} = -10.29 + 6.62 = -3.67$$

This represents the bending moment at $x = 0.2l$ in the continuous truss for a unit value of H. The values of $\frac{H}{P}(m_h + m_{th})$ are calculated in Columns 2 and 6 of Table 5 by multiplying the H-influence line ordinates by -3.67.

Values of m_p , as shown in Fig. 8(b) are tabulated in Column 7 of Table 5, and are the same as were calculated in Column 8 of Table 2 for the non-continuous structure. The angles ϕ_{p1} and ϕ_{p2} shown in Fig. 8(b) are calculated by differentiating Eqs. 8 and 9, with $\phi_{p1} = 0.2466$ and $\phi_{p2} = 0.0035$. These angles must be equated to the angles produced by T_{p1} and T_{p2} , obtained from Table 4.

$$\begin{aligned}\phi_{p1} &= 2.506 \frac{T_{p1}}{H_r l_1} + 6.000 \frac{T_{p1}}{H_r l} + 0.987 \frac{T_{p2}}{H_r l} = 0.2466 \\ \phi_{p2} &= 2.506 \frac{T_{p2}}{H_r l_1} + 6.000 \frac{T_{p2}}{H_r l} + 0.987 \frac{T_{p1}}{H_r l} = 0.0035\end{aligned}$$

Solving these equations simultaneously yields $T_{p1} = 0.02254 H_r l$ and $T_{p2} = -0.00171 H_r l$. The values of m_{tp} in the side spans are calculated in Column 3 of Table 5, by multiplying values in Column 2 of Table 4 by $-0.02254 l$ or -18.03 for the left side and $+0.00171 l$ or 1.37 for the right side span. The values of m_{tp} in the main span are calculated separately for T_{p1} and T_{p2} . Column 8 of Table 4 is multiplied by $-0.02254 l$ to yield Column 8 of Table 5, and Column 7 of Table 4 is multiplied by $+0.00171 l$ to yield Column 9 of Table 5. The final bending moment influence line is obtained for the side spans in Column 4 by combining Columns 2 and 3. The influence line for the main span is obtained in Column 10 as the sum of ordinates in Columns 6, 7, 8, and 9.

The temperature stresses are calculated in a similar manner to those for the simply supported structure.

$$H_t = + \frac{\omega t L_t}{D_1} = + \frac{0.0000065 \times 60 \times 1998}{0.006506} = + 119.9 \text{ kips.}$$

The change in bending moment resulting from the temperature change has the following value at $x = 0.2l$

$$M_t = H_t (m_h + m_{th}) = \pm 119.9 \times 3.67 = \pm 440 \text{ ft-kips.}$$

The final values of H and M will be calculated for the uniform live load extending from the left tower to $x = 0.41l$ of the main span in order to compare results with those of Dr. Steinman¹⁵. Another set of influence lines was calculated for $c/l = 6.00$ and areas were obtained by Simpson's rule. The final values for $c/l = 6.632$ (used by Steinman) are interpolated. The final bending moment for $c/l = 7.0$ is 6570 ft-kips and for $c/l = 6.0$ is 7480 ft-kips. The interpolation yields a bending moment of 6900 ft-kips as compared to 6970 ft-kips calculated by the deflection theory¹⁶. Some of the error is probably computational error.

CONCLUSIONS

Long-span suspension bridges may be readily analyzed by the use of influence lines. Two sets of influence lines are necessary, corresponding to different values of c/l . A linear interpolation yields the desired accuracy. For each set of influence lines, c/l is assumed constant. This permits superposition of loads, bending moments, shears, and deflections. Any type of continuous or non-continuous structure may be considered.

Influence lines for shear may be constructed in the same manner as the bending moment influence lines. The influence line for shear in a simple beam in tension is obtained by differentiating Eqs. 8 and 9. The shear effect of the cable is found by differentiating Eq. 10. The final influence line for shear in a non-continuous bridge is then obtained by multiplying the H -influence line ordinates by a constant, and superimposing them on the influence line for a simple beam in tension. In a continuous structure, the continuity effects are also superimposed.

The influence line procedure has many advantages over more abstract mathematical procedures. The designer can visualize the shape of the influence lines as deflection curves of the structure, and thus check each step of his work against gross errors. The calculation work is facilitated by use of tabulated ordinates of deflection curves in dimensionless form.

ACKNOWLEDGMENT

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15. Ibid., Footnote 2, page 2.

16. Ibid., Footnote 1, page 1.

17. Ibid., Footnote 10, page 4.

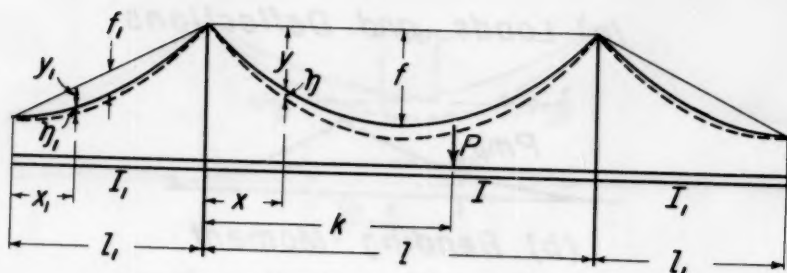


Fig. 1 Notation Diagram

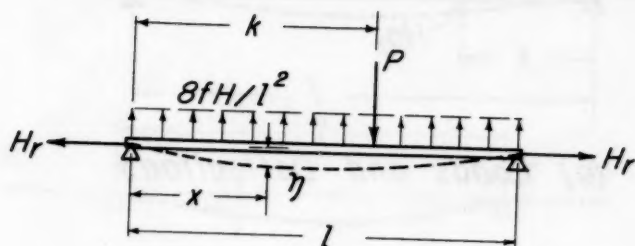
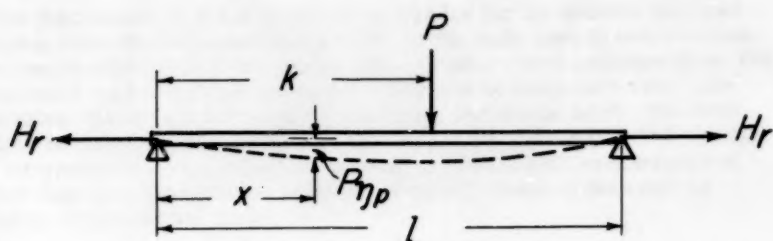
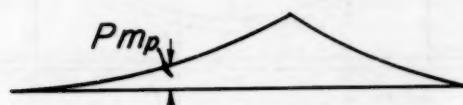


Fig. 2 Analogous Beam

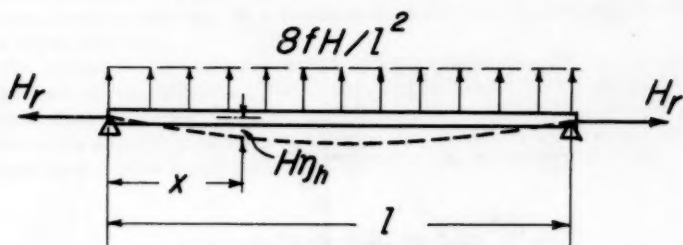


(a) Loads and Deflections

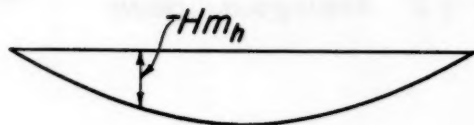


(b) Bending Moment†

Fig. 3 Beam with Live Load



(a) Loads and Deflections



(b) Bending Moment

Fig. 4 Beam with Suspender Loads

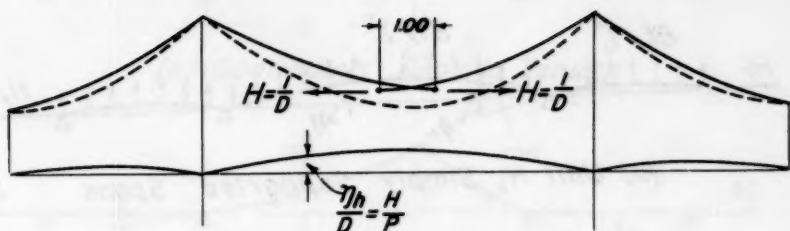


Fig. 5 Deflection Curve Representing Influence Line for H

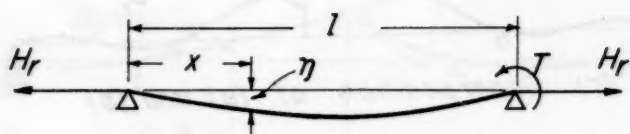
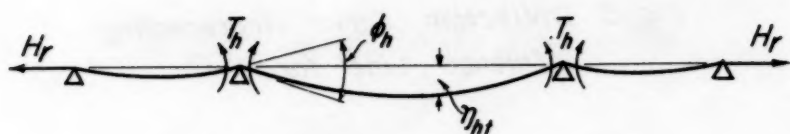


Fig. 6 Beam with End Moment



(a) Unit H , Simply Supported Spans

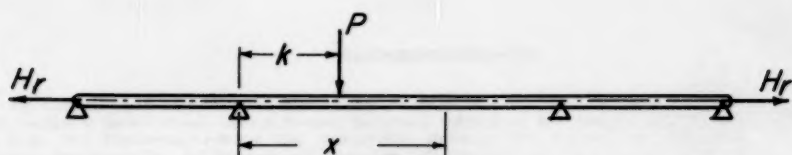


(b) Corrections for Continuity

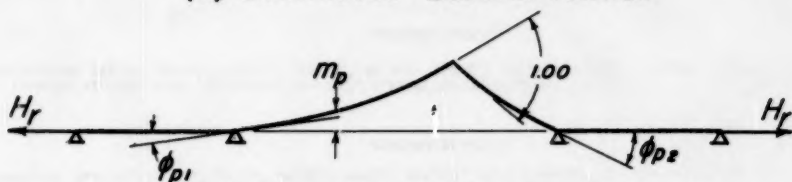


(c) Superposition of (a) and (b)

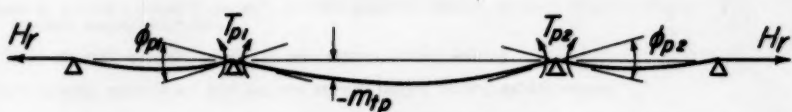
Fig.7 Influence Line for H



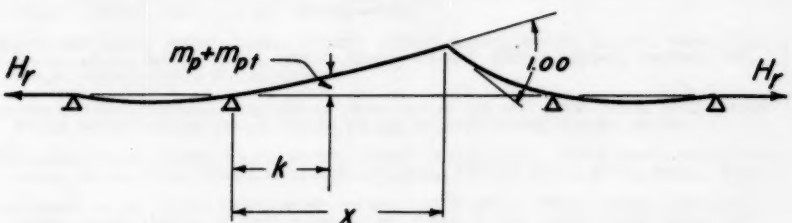
(a) Continuous Beam in Tension



(b) Simple Beam in Tension



(c) Effects of Continuity



(d) Superposition of (b) and (c)

Fig. 8 Influence Line for Beam Bending Moment



FIGURE 1. THE STANDARD CURVE



FIGURE 2. THE STANDARD CURVE



FIGURE 3. THE STANDARD CURVE

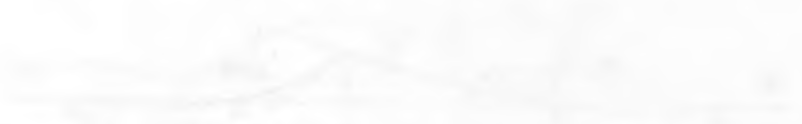


FIGURE 4. THE STANDARD CURVE

THE STANDARD CURVE

THE STANDARD CURVE

THE STANDARD CURVE

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VOLUME 80 (1954)

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JUNE: 444(SM)^e, 445(SM)^e, 446(ST)^e, 447(ST)^e, 448(ST)^e, 449(ST)^e, 450(ST)^e, 451(ST)^e, 452(SA)^e, 453(SA)^e, 454(SA)^e, 455(SA)^e, 456(SM)^e.

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AUGUST: 466(HY), 467(HY), 468(ST), 469(ST), 470(ST), 471(SA), 472(SA), 473(SA), 474(SA), 475(SM), 476(SM), 477(SM), 478(SM)^c, 479(HY)^c, 480(ST)^c, 481(SA)^c, 482(HY), 483(HY).

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OCTOBER: 512(SM), 513(SM), 514(SM), 515(SM), 516(SM), 517(PO), 518(SM)^c, 519(IR), 520(IR), 521(IR), 522(IR)^c, 523(AT)^c, 524(SU), 525(SU)^c, 526(EM), 527(EM), 528(EM), 529(EM), 530(EM)^c, 531(EM), 532(EM)^c, 533(PO).

NOVEMBER: 534(HY), 535(HY), 536(HY), 537(HY), 538(HY)^c, 539(ST), 540(ST), 541(ST), 542(ST), 543(ST), 544(ST), 545(SA), 546(SA), 547(SA), 548(SM), 549(SM), 550(SM), 551(SM), 552(SA), 553(SM)^c, 554(SA), 555(SA), 556(SA), 557(SA).

DECEMBER: 558(ST), 559(ST), 560(ST), 561(ST), 562(ST), 563(ST)^c, 564(HY), 565(HY), 566(HY), 567(HY), 568(HY)^c, 569(SM), 570(SM), 571(SM), 572(SM)^c, 573(SM)^c, 574(SU), 575(SU), 576(SU), 577(SU), 578(HY), 579(ST), 580(SU), 581(SU), 582(Index).

c. Discussion of several papers, grouped by Divisions.

d. Presented at the Atlanta (Ga.) Convention of the Society in February, 1954.

e. Presented at the Atlantic City (N.J.) Convention in June, 1954.

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ERNEST W. CARLTON

OLIVER W. HARTWELL

DON M. CORBETT